Biplot Simulation of Exponential Function to Determine Body Dimensions' Growth Rate of Bali Calf

Putu Sampurna, Ketut Saka, Lanang Oka, Sentana Putra

Abstract - Biplot simulation of exponential function was performed to determine body dimensions'growth rate of bali calf using biplot analysis with Promax rotation (90). Biplot graphic demonstration showed that exponential functions with slow growth occurred in quadrant II, with medium growth occurred in quadrant I, and with fast growth occurred in quadrant IV. Biplot demonstration of the body dimensions in male and female Bali calves showed 3 pieces of quadrants i.e. quadrant I, quadrant II and quadrant IV of two-dimensional eigenspace. Based on the result, it can be determined that the body dimensions Bali calves were as follows; in quadrant II was at a slow growth rate, in quadrant I was at a medium growth rate, and in quadrant IV was at a rapid growth rate

Keywords- Simulation. exponential function, biplot, eigen vector, quadrant, Bali calves, body dimensions and the growth rate

I. INTRODUCTION

The growth of the animal's body dimensions during their rapid growth usually follows an exponential function with growth rate varies between one to another body dimension. The differences in growth rate are due to differences in physiology, functional demands and the components of bodies. The dimensions of the animal's body that functions early, will grow earlier than its growth rate and is therefore greater than the dimensions of the animal's body that function later. For instance, the dimensions of the body composed mainly of bones will develop earlier than those composed of muscle or fat (Swatlan, 1984), Sampurna and Suata, 2008).

Growth of organs, tissues and other parts of animals' bodies at the time of their rapid growth usually follows exponential function with growth rates vary from one to another body dimensions.

Differences in growth rate is caused by the differences in the functional demands of different organs, tissues, or body parts of animals and those which grow up first will then have the growth rate greater than the organ, tissue, or body parts of animals that grow up later.

The differences in the growth rates among various animal's body dimensions from birth until a certain age can be determined using the regression line equation $Y = a\, e^{kx}$, where Y is the value of the animal's body dimensions, a is the size at birth, k is the growth rate, X is the age of the animals and e is the natural logarithm 2.71828 ...... Based on the regression line equation, the value of k varies among various body dimensions

In many studies, however, researchers are often faced with too many body dimensions, so the presentation in the form of an exponential function is less attractive, or can not be presented in one image simultaneously. Therefore, the presentation of growth rate differences among various body dimensions will be demonstrated using a cross axis eigenvectors in two-dimensional space

In the multipariate analysis of multiple variables, the eigen value structure of data matrix is crucially important. Eigen structure is expressed in the root of eigen values and eigen vectors. Observation matrix consisting of n objects each with p variables, is transformed into $Z = X\varphi$, with $\Phi = (\varphi 1, \varphi 2, ..., \varphi p)$ is a pxp orthogonal matrix and φi is the eigen vector of X'X.

A square matrix of order nxn, let A, and a column vector X. Vector X is a vector in space Euklidian associated with an equation: Where is a scalar and X is a non-zero vector scalar called Eigen values of the matrix A. Eigen values are the characteristic values of a square matrix.

$A \in R^{nxn}$ matrix is symmetrical, if the A′A, and is orthogonal matrix $A \in R^{nxn}$ if AA′ = A′A = I. It is clear that A is nonsingular with A-1 = A′

$A \in R^{nxn}$ matrix, A = (a_{ij}) is a diagonal matrix if a_{ij} = 0, i ≠ j. A diagonal matrix can also be written as A = diag $(a_1, a_2,........, a_n)$, with $a_1, a_2,........, a_n$ are the elements of the main diagonal.

A symmetrical matrix can be decomposed into A = VDV′, with V as an orthogonal matrix and D a diagonal matrix.

The length (norm) vector u in the space R^n, if u = $(u_1, u_2, u_3, ..., u_n)$, then the length is:

$$|\bar{u}| = \sqrt{u_1^2 + u_2^2 + u_3^2 + ... + u_n^2}$$

and if the vectors u and v vectors in space R^n, then the distance between vectors u and v are:

$$d(u,v) = \sqrt{(u_1 - v_1)^2 + (u_2 - v_2)^2 + (u_3 - v_3)^2 + ... + (u_n - v_n)^2}$$

Biplot is an exploratory analysis of multiple variable data that can be illustrated in a graphic picture of the closeness among observed objects, the diversity of variables, the correlation among the variables, and the relationship between variables and the object. In addition, biplot analysis is used to describe the relationship between variables and objects that are at a high-dimensional space into two-dimensional space.

Table 1: Data from Exponential Function

x	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9	Y10	Y11	Y12
0	2.0	2.0	8.0	2.0	2.0	2.0	2.0	8.0	2.0	8.0	2.0	8.0
1	2.0	2.1	8.5	2.2	2.7	3.0	3.3	14.6	6.6	32.4	6.6	32.4
2	2.1	2.2	9.0	2.4	3.6	4.5	5.4	26.6	22.1	131.6	22.1	131.6
3	2.1	2.3	9.6	2.5	4.9	6.6	9.0	48.4	73.2	533.5	73.2	533.5
4	2.2	2.4	10.2	2.8	6.6	9.9	14.8	88.2	243.0	2163.4	243.0	2163.4
5	2.2	2.4	10.8	3.0	9.0	14.8	24.4	160.7	806.9	8773.0	806.9	8773.0
6	2.3	2.5	11.5	3.2	12.1	22.1	40.2	292.8	2678.9	35576.3	2678.9	35576.3
7	2.3	2.7	11.2	3.5	16.3	32.9	66.2	533.5	8894.1	144269.0	8894.1	144269.0
8	2.4	2.8	12.9	3.8	22.1	49.1	109.2	972.1	29529.4	585039.1	29529.4	585039.1
9	2.4	2.9	13.7	4.1	29.8	73.2	180.0	1771.2	98040.9	2372448.4	98040.9	2372448.4
10	2.4	3.0	14.6	4.5	40.2	109.2	296.8	3227.4	325507.0	9620741.7	325507.0	9620741.7
11	2.5	3.1	15.5	4.8	54.2	162.9	489.4	5880.7	1080720.3	39014002.7	1080720.3	39014002.7
12	2.5	3.2	16.4	5.2	73.2	243.0	806.9	10715.4	3588114.8	158209483.4	3588114.8	158209483.4
13	2.6	3.4	17.5	5.7	98.8	362.5	1330.3	19524.7	11912951.0	641570284.9	11912951.0	641570284.9
14	2.7	3.5	18.5	6.1	133.4	540.9	2193.3	35576.3	39552358.4	2601693048.0	39552358.4	2601693048.0
15	2.7	3.6	19.7	6.6	180.0	806.9	3616.1	64824.3	131318348.3	10550376843.3	131318348.3	10550376843.3

Three approaches of matrix associated with the data, variables, and objects, can be obtained from biplot. Size suitability of the three matrices was presented by Gabriel in 2002.

In the interpretation of the relevant factors, the first selection step is generally followed by a rotation of maintained factor which serves to make the output more understandable. The rotation can be orthogonal or oblique rotation (which allows correlated factors), Promax rotation is an alternative non-orthogonal (oblique) rotation method which can compute faster than direct oblimin method and it is, therefore sometimes used to compute a very large dataset. In practice rotation of factor analysis, it is therefore advisable to try several sizes to subspaces of retained factors to assess the robustness of interpretation rotation (Abdi, 2003).

Information which can be obtained from the biplot in two-dimensional space of the exponential function is to describe the close relationship on the basis of its rates. Two exponential functions with almost the same rate are described as two adjacent point position, whereas those with a great different rate are described as a two distant point position. This information will then be used to determine the differences in the growth rates of Bali cattle body dimensions, whether the growth of a particular body dimension is slow, medium or fas

II. METHODS

Data were generated from the 12 pieces of the exponential functions of $Y_i = a\, e^{kx}$, at many different rates (k) and the values, but with the same X of 0, 1, 2, 3. 4. 5. 6. 7, 8, 9, 10, 11, 12, 13, 14, and 15, Y_1: a = 2. k = 0.02, Y_2: a = 2. k = 0.04, Y_3:

a = 8. k = 0.06, Y_4: a = 2. k = 0:08, Y_5: a = 2. k = 0.3, Y_6: a = 2. k = 0.4, Y_7: a = 2. k = 0.5, Y_8: a = 8. k = 0.6, Y_9: a = 2. k = 1.2, Y_{10}: a = 8. k = 1.4, Y_{11}: a = 2. k = 1.6, and Y_{12}: a = 2. k = 1.8 to obtain a chest in Table 1.

Data were analyzed by factor analysis based on the correlation between variables, in order to obtain the correlation matrix and their significant differences. Eigen vectors are determined by the relationship $\mathbf{R x_o} = \lambda\, \mathbf{x_o}$, so that $\mathbf{f(x)} = \left| \mathbf{R} - \lambda\, \mathbf{I} \right| = \mathbf{0}$. So $\mathbf{f(\lambda) = 0}$ is called R matrix eigen value equation, the roots of this equation is called the root of the matrix R eigen values, and root vectors compatible with eigen values is called eigenvectors.

Prior to analysis, the data obtained from the exponential function were divided by their average, so that all data on each exponential function have different rates, all averages are 1.0, and differ only in their standard deviation.

Demonstration biplot graphs drown from two eigenvectors corresponding to the largest eigen value roots as X-axis and the second largest eigen roots as Y-axis , with Promax Rotation (90).

The coordinate position of each exponential function on X and Y determines the close relationship between the exponential functions. Exponential functions with almost the same rates will be at close position, whereas those with difference rate will be at a distant position. Promax Rotation (90) is used to divide the location of the exponential function in the cross-axis two-dimensional eigenspace (biplot) into 3 points (quadrant). Exponential functions located in quadrant II is an exponential function of slow rate, those in quadrant I is an exponential function of the medium rate, and those quadrant IV is an exponential function of fast rates.

Biplot analysis was used to analysed body dimensions of Bali calves aged 0 months to 9 months raised in the village Getasan, District Petang, District Badung, Bali. The data were taken every month by measuring 6 male and 6 female bali calf. Data on body dimensions were collected by measuring the lengths of heads, necks, bodies, hears, tails, horns, upper flank, lower hind legs, middle hind legs. upper hind legs, upper body, lower front legs, middle front legs, and upper front legs. The circumferences of chest, waist, front and neck. Other data collected were depth of chest, waist and neck, as well as the width of head, neck, cheeks, chest, waist, and ass. The collected data were analysed by using SPSS (Statistical Product and Service Solutions) program.

RESULTS AND DISCUSSION

The results of factor analysis (Table 2) showed that 98 777% of the 12 exponential functions which have a different rate, can be described in the biplot two-dimensional

eigenspace, and Scree Plot (Figure 1) shows that a sharp decline in values was observed on the root of eigen values at their first and second principal components. The use of two main components are therefore considered to be able to explain the diversity of the exponential function data with many different rates.

Table 2: Total Variance Explained Exponential Function

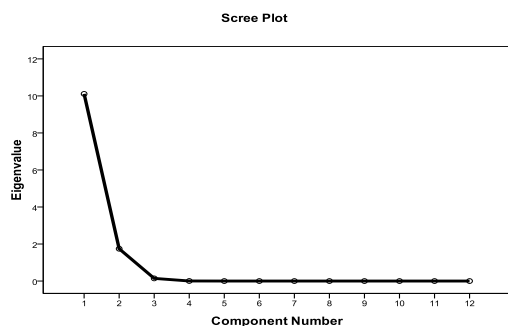| Component	Initial Eigenvalues		
	Total	% of Variance	Cumulative %
1	10.109	84.242	84.242
2	1.744	14.535	98.777
3	.143	1.192	99.969
4	.003	.029	99.998
5	.000	.002	100.000

Figure 1: Sree Plot Exponential Function at a Different Rate

Table 3 and Figure 2, shows that the exponential function with a rate (k = 0:02 to 0:08) has a negative abscissa and a positive ordinate which fall into quadrant II, an exponential function classified as slow pace. Functions that have exponential rate (k = 0.3 to 0.6 has a positive abscissa and a positive ordinate which fall into quadrant I, an exponential function classified as medium pace. Functions that have exponential rate (k = 1.2 to1.8) has a positive abscissa and negative ordinate which fall into quadrant IV, the exponential function classified into a fast rate.

Table 3: Pattern Matrix Exponential Function

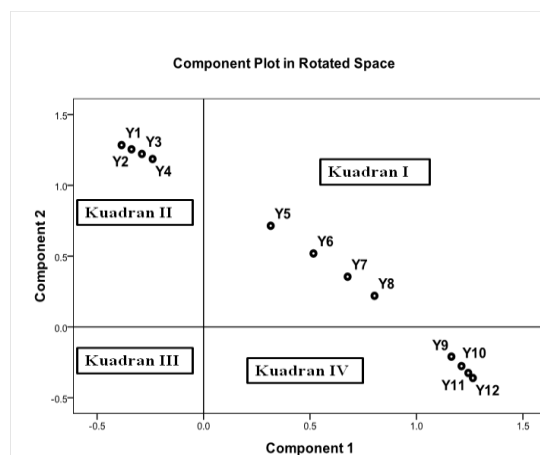| Variable	Component		Quadrant
	1	2	
Y1 : a=2 k=0.02	-.385	1.285	II
Y2 : a=2 k=0.04	-.339	1.255	II
Y3 : a=8 k=0.06	-.290	1.222	II
Y4 : a=2 k=0.08	-.239	1.186	II
Y5 : a=2 k=0.3	.315	.715	I
Y6 : a=2 k=0.4	.516	.519	I
Y7 : a=2 k=0.5	.676	.355	I
Y8 : a=8 k=0.6	.803	.220	I
Y9 : a=2 k=1.2	1.164	-.209	IV
Y10: a=8 k=1.4	1.211	-.276	IV
Y11: a=2 k=1.6	1.243	-.325	IV
Y12: a=2 k=1.8	1.265	-.360	IV

Figure 2: Eigen Vectors Exponential Function at a Different Rate

The results of the factor analysis (Table 4) on Bali male calves showed that 96.83% body dimensions can be visualized on two-dimensional biplot eigenspace, and Scree Plot (Figure 3) shows that a sharp decline in values was observed on the root of eigen values at their first and second principal components. The use of the two main components are therefore considered to be able to explain the diversity of data on male bali calf body dimensions

Table 4: Total Variance Explained Body Dimension Calf Bali Males

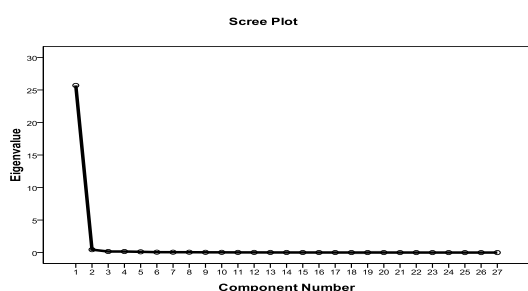| Component	Initial Eigenvalues		
	Total	% of Variance	Cumulative %
1	25.576	94.725	94.725
2	.499	1.850	96.575
3	.277	1.026	97.600
4	.138	.510	98.110

Figure 3: Sree Plot Body Dimension Calf Bali Males

It appears from Table 5. and Figure 4 that the body dimensions of slow growth rates was in quadrant II, consisting of the length of horns and ear, and the depth of the chest respectively from the slowest and the fastest growth on this quadrant. The body dimensions with medium growth rate which was in quadrant I consisting of the width neck, the length of the body and the last is the length of the neck respectively from the slowest to the fastest on this quadrant. The body dimensions with fast growth rate which was in quadrant IV, consisted of high flank, the middle front legs and lower front legs respectively from the s the fastest and the slowest growth rates in this quadrant

Table 5: Pattern Matrix Body Dimension Calf Bali Males

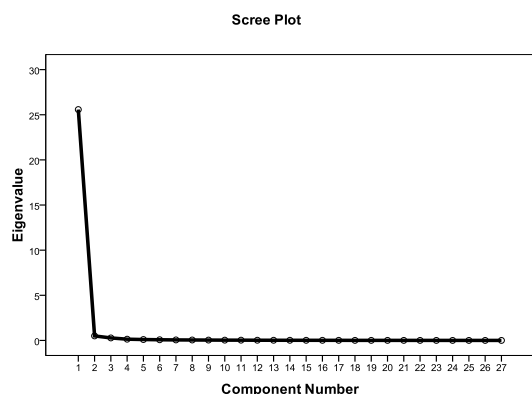| Quadrant	Body Gimension	Component	
		1	2
II	Length Horns	-1.966	2.903
II	Length Hears	-1.050	2.014
II	Width Waist	-0.636	1.612
II	Length Tail	-0.256	1.241
II	Length Front Legs Section Over	-0.242	1.222
II	Depth of Neck	-0.158	1.140
I	Width Neck	0.223	0.757
I	Length Body	0.334	0.659
I	Width Chest	0.417	0.568
I	Length Head	0.465	0.531
I	Width Cheeks	0.582	0.404
I	Circumference Neck Real	0.605	0.385
I	Circumference Neck Front	0.706	0.284
I	Width Head	0.721	0.252
I	Circumference Chest	0.779	0.206
I	Length Hind Legs Section Middle	0.882	0.100
I	Length Neck	0.972	0.020
IV	Length Front Legs Section Below	0.995	-0.016
IV	Length Hind Legs Section Over	1.059	-0.075
IV	Depth of Chest	1.137	-0.154
IV	Circumference Waist	1.162	-0.178
IV	Width Ass	1.625	-0.648
IV	Depth of Waist	1.694	-0.723
IV	High Body	1.815	-0.846
IV	Length Hind Legs Section Below	1.882	-0.930
IV	Length Front Legs Section Middle	1.949	-0.994
IV	High Flank	2.017	-1.054
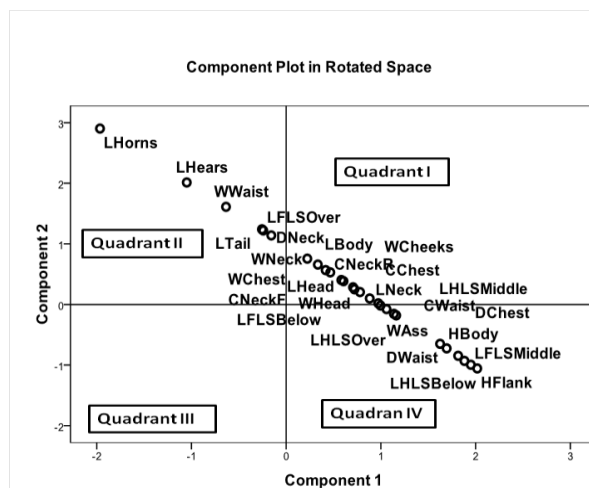
Figure 4: Eigen Vectors Body Dimension Calf Bali Males

The results of the factor analysis (Table 6) on Bali female calves showed that 96.58% body dimensions can be described in two-dimensional biplot eigenspace, and Scree Plot (Figure 5) shows that a sharp decline in values was observed on the root of eigen values at their first and second principal components. The use of the two main components are therefore considered to be able to explain the diversity of data on female bali calf body dimensions

Table 6: Total Variance Explained Body Dimension Calf Bali Females

Component	Initial Eigenvalues		
	Total	% of Variance	Cumulative %
1	25.576	94.725	94.725
2	.499	1.850	96.575
3	.277	1.026	97.600
4	.138	.510	98.110

Figure 5: Sree Plot Body Dimension Calf Bali Females

Table 7: Pattern Matrix Body Dimension Calf Bali Females

Quadrant	Body Dimension	Component	
		1	2
II	Length Horns	-2.139	3.049
II	Length Hears	-0.761	1.733
II	Length Tail	-0.340	1.320
II	Depth of Neck	-0.287	1.270
II	Width Waist	-0.191	1.162
II	Length Front Legs Section Middle	-0.012	0.994
I	Length Head	0.246	0.744
I	Length Front Legs Section Below	0.260	0.719
I	Depth of Chest	0.377	0.611
I	Length Hind Legs Section Middle	0.390	0.599
I	Length Body	0.446	0.541
I	Length Hind Legs Section	0.541	0.448
I	Circumference Neck Front	0.561	0.429
I	Depth of Waist	0.580	0.413
I	Width Cheeks	0.899	0.092
I	Length Front Legs Section Over	0.943	0.039
IV	Circumference Chest	1.028	-0.043
IV	Width Neck	1.150	-0.208
IV	Circumference Waist	1.285	-0.300
IV	Width Chest	1.490	-0.514
IV	High Body	1.495	-0.519
IV	Circumference Neck Real	1.590	-0.621
IV	Length Neck	1.624	-0.656
IV	High Flank	1.727	-0.746
IV	Length Hind Legs Section Below	1.766	-0.799
IV	Width Head	1.926	-0.986
IV	Width Ass	1.938	-0.971
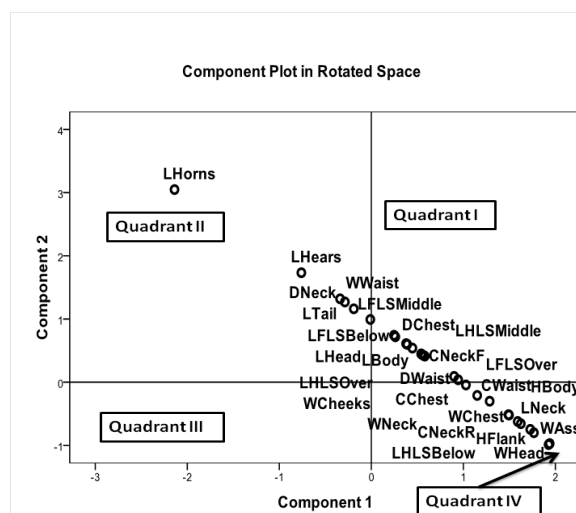
Figure 6: Eigen Vectors Body Dimension Calf Bali Females

It appears from Table 7. and Figure 6. that the body dimensions of slow growth rate was in quadrant II consisting of the length of horns, the length of ear, and the length of middle front legs respectively from the slowest to the fastest growth in this quadrant. The body dimensions of medium growth rate were in quadrant I consisting of the length of head, the length lower front legs, and the length upper front legs respectively from the slowest to the fastest. The body dimensions with fast growth rate was in quadrant IV, consisting of width of ass, the width of the head, and circumference of the chest respectively from the fastest to the slowest.

The simulation of biplot exponential functions showed that exponential functions which have a slow rate (k = 0:02 to 0:08), was in quadrant II, those which have medium rate (k = 0.3 to 0.6) was in quadrant I, and those which have fast rate rapid (k = 1.2 to1.8) was in quadrant IV. Biplot analysis on body dimensions of male and female bali calves, also located in quadrant II, quadrant I and quadrant IV. Based on the simulation of biplot exponential function, it can be concluded that body dimensions of bali calf barrel-chested in quadrant II were those with slow growth rate, barrel-chested in quadrant I were those with medium growth rate, ande barrel-chested in quadrant IV were those with a fast growth rate.

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BIOGRAPHIES

Putu Sampurna got Engineering degree in Faculty Husbandry, University of Udayana 1983, Bridging Programe Mathematics ITB 1988, Master of Science in Applied Statistics IPB 1992, Students doctoral program Animal Husbandry at the university Udayana. He is a lecturer at the Laboratory of Biostatistics, Faculty of Veterinary Medicine, Udayana University.

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